Algebra 1 Notes SOL A.2 (9.8, 9.6) Factor By Grouping and $ax^2 + bx + c$ Mrs. Grieser

Name: _____ Block: _____ Date: _____

Factoring By Grouping

Review Type I Factoring: Factor out greatest common monomial factor

Review Type II Factoring: Factor difference of squares (sum and difference pattern)

Review Type III Factoring: Factor $x^2 + bx + c$

Type IV Factoring – Four Terms: Factor by Grouping

If we are given a four term polynomial, we split the polynomial into two sets of two terms, and factor those sets using type I factoring. If we find a common polynomial, we use type I factoring again to factor it out.

Factoring a common polynomial: Factor $x(x - 5) + 3(x - 5)$

Notice there is a common polynomial of $x - 5$. We use type I factoring to factor it out, and are left with $x + 3$. So the factored form is $(x - 5)(x + 3)$.

Examples:

a) $5x^2(x - 2) + 3(x - 2)$ b) $7y(5 - y) - 3(y - 5)$ c) $11x(x - 8) + 3(8 - x)$
(Factor out -1 to make signs match!)

We use this skill to factor a four term polynomial. Factor the first two terms, then factor the second two terms. Then factor the common polynomial.

Examples:

a) $n^3 + 6n^2 + 5n + 30$
$= (n^3 + 6n^2) + (5n + 30)$
$= n^2(n + 6) + 5(n + 6)$
$= (n + 6)(n^2 + 5)$

b) $m^3 + 7m^2 - 2m - 14$
$= (m^3 + 7m^2) + (-2m - 14)$
$= m^2(m + 7) - 2(m + 7)$
$= (m + 7)(m^2 - 2)$

c) $9x^3 + 9x^2 - 7x - 7$

You try: Factor the expression

a) $2x(x + 4) - 3(x + 4)$ b) $3y^2(y - 2) + 5(2 - y)$ c) $x^3 + 3x^2 + 5x + 15$

d) $x^3 + x^2 + x + 1$ e) $y^2 + y + yx + x$ f) $x^3 - 6 + 2x - 3x^2$
(HINT: Rearrange terms in degree order!)

Algebra 1 Notes SOL A.2 (9.8, 9.6) Factor By Grouping and $ax^2 + bx + c$ Mrs. Grieser

Type V Factoring – Factor $ax^2 + bx + c$

We can factor polynomials of the form $x^2 + bx + c$ (type III factoring). What do we do to factor polynomials of this form when the leading coefficient is not 1?

Guess and Check

Factor $2x^2 - 7x + 3$

- Draw sets of parentheses: ( )( )
- In this case, the first terms in each must be 2x and x (why?) and the signs must be negative (why?): (2x -)(x -)
- The factors of 3 are 1 and 3; test by multiplying back to see what works
  - $(2x - 3)(x - 1) \longrightarrow 2x^2 - 5x + 3$ NOPE!
  - $(2x - 1)(x - 3) \longrightarrow 2x^2 - 7x + 3$ YES!!
- Factors are $(2x - 1)(x - 3)$

Factor by Grouping Method

If you are not a good guesser, it can be hard sometimes to use the guess and check method. We can use what we know about factoring by grouping to help us.

Factor $15x^2 + 13x + 2$

- Multiply a x c (15 x 2 = 30)
- What factors of 30 add to 13? (10 and 3)
- Split up middle term: $15x^2 + 10x + 3x + 2$
- Group: $5x(3x + 2) + (3x + 2)$
- Factor out polynomial: $(3x + 2)(5x + 1)$
- VERIFY (do not skip this step): $(3x + 2)(5x + 1) = 15x^2 + 13x + 2$ ✓

Sometimes the terms have a common factor. FACTOR OUT THE GCF BEFORE PROCEEDING!!

Examples:

a) $6x^2 - 11x - 10$

b) $3x^2 + 14x - 5$

c) $4x^2 + 26x - 14$

You try: Factor the polynomials

a) $3x^2 + 8x + 4$

b) $4x^2 - 9x + 5$

c) $2x^2 - 13x + 6$

d) $-4x^2 + 12x + 7$
(hint: factor out -1 first)

e) $4x^2 + 11x - 3$

f) $12x^2 - x - 6$