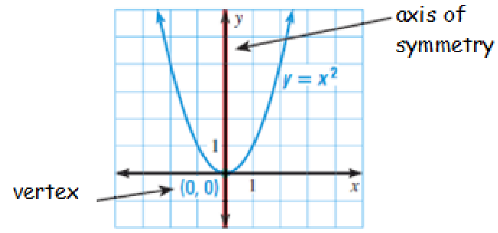
Name: _________________________________ Block: _______ Date: ______________

Quadratic Equations

We've worked with quadratic equations when we factored them in chapter 9. We will learn more about them here, as well as a formula for finding zeros and the number of zeros one may have.

- Quadratic Function Standard Form: $ax^2 + bx + c$, where $a \neq 0$.
- Parent Function: $y = x^2$
- The graph of a quadratic is a parabola.
- The vertex of the parabola is the lowest or highest point of the parabola.
- The axis of symmetry is a straight line that passes through the vertex dividing the parabola into two symmetric parts.



- When the leading coefficient a is positive, the parabola opens up ("concave up"); when a is negative, the parabola opens down ("concave down").
    - When a parabola is concave up, it will have a minimum value.
    - When it is concave down, it will have a maximum value.
    - The minimum or maximum value is the y-value of the vertex.
- When $|a| > 1$, the parabola is narrower (gets skinny). When $|a| < 1$, the parabola is wider.
- The y-intercept of a parabola is c (occurs when x = 0).
- The value b helps us find the value of the vertex: the x-coordinate of the vertex is $-\frac{b}{2a}$.

Solving Quadratic Equations

We solved quadratic equations algebraically in chapter 9. We arrange the terms such that they are all on one side and 0 on the other (we put it in standard form). Then we factored and used the zero product property to find the zeros of the function.

We can also solve quadratic functions graphically. Graph the function manually or by using a graphing calculator. The zeros of the function are the x-intercepts, where the curve touches or crosses the x-axis.

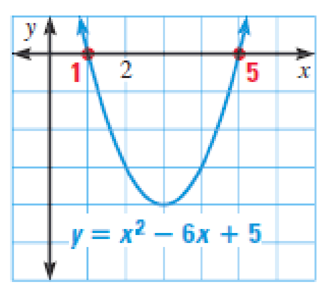
Example: Solve $x^2 - 6x + 5 = 0$

Solve Algebraically:

$x^2 - 6x + 5 = 0$	Make sure in standard form
$(x - 5)(x - 1) = 0$	Factor
$x = 5, x = 1$	Zero product property

Solve Graphically:

- Graph using y= and graph buttons on the graphing calculator.
- Eyeball where curve touches or crosses the x-axis, or use table, trace, or zeros function to find where y = 0.



The Quadratic Formula

There is another way to find the zeros (roots) of a quadratic equation! The solutions to $ax^2+bx+c = 0$ may be found by plugging the a, b, and c values into the quadratic formula:

The Quadratic Formula
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(Whew! It's on the SOL formula sheet!!)

Example: Find the zeros of $f(x) = x^2 - 6x + 5$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{6^2 - 4(1)(5)}}{2(1)} = \frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2} = 5 \text{ or } 1$$

You try: Use the quadratic formula to find the zeros of these functions…

a) $f(x) = x^2 + 5x - 104$ b) $f(x) = 4x^2 - 7x - 2$ c) $f(x) = x^2 - 8x + 16$

Number of Zeros

As we can see in the previous example, sometimes we saw two solutions (the curve crossed the x-axis twice), and sometimes we saw one solution (the curve touched the x-axis once). Sometimes there are no solutions, as the curve NEVER touches or crosses the x-axis (graph $f(x) = x^2 + 2$ for an example). We can use the radicand part of the quadratic formula (called the discriminant) to tell us the number of solutions.

The discriminant of the quadratic formula is $b^2 - 4ac$.

When the disriminant is:

- greater than 0, we have two real solutions (we will add and subtract its radical)
- = 0, we have one real solution (we will get 0 for the radical)
- less than zero, we have no real solutions (we will get a negative number under the radical, unsolvable in the real number system)

Example: How many solutions are there for…

a) $x^2 + 4x + 3 = 0$ b) $2x^2 - 5x + 6 = 0$ c) $-x^2 + 2x = 1$

You try: Find the number of zeros for the following quadratic functions. Then find the zeros (if there are any):

a) $y = x^2 + 10x + 25$ b) $f(x) = x^2 - 9x$ c) $f(x) = -x^2 + 2x - 4$