Name: $\qquad$ Block: $\qquad$ Date: $\qquad$

## Slope

The slope of a line is the ratio of the vertical change (the "rise") to the horizontal change (the "run") between any two points in a line. The letter ' $m$ ' is used to denote slope.

Looking at a graph of a line, we can find the line's slope by first identifying two points. We can see that the "rise" is the same as the difference in the ordinates ( $y$-values) of the two points and that the "run" is the same as the difference in the abscissas (x-values) of the two points.
To calculate slope for two points ( $x_{1}, y_{1}$ ) and ( $x_{2}, y_{2}$ ):


$$
\text { slope }=m=\frac{\text { change in } y}{\text { change in } x}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{\Delta y}{\Delta x}
$$

The Greek letter delta ( $\Delta$ ) is used to denote the difference between two values and may be used as a short cut $-\Delta y$ is shorthand for $y_{2}-y_{1}$.
Examples - find the slopes given the following sets of points:
a) $(-4,2)$ and $(2,6)$
b) $(3,5)$ and $(6,-1)$
c) $(-2,4)$ and $(4,4)$
$x_{1}=-4, y_{1}=2$
$x_{1}=3, y_{1}=5$
$x_{1}=-2, y_{1}=4$
$x_{2}=2, y_{2}=6$
$x_{2}=6, y_{2}=-1$
$x_{2}=4, y_{2}=4$
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{6-2}{2-(-4)}=\frac{4}{6}=\frac{2}{3}$
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{-1-5}{6-3}=\frac{-6}{3}=-2$
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{4-4}{4-(-2)}=\frac{0}{6}=0$




Things to know about slope:

- Positive slopes: range increases as $\times$ increases - line rises from left to right.
- Negative slopes: range decreases as $x$ increases - line falls from left to right.
- $\underline{0}$ slope: horizontal lines have a slope of 0 because the difference of their $y$-values is always 0 .
- Undefined slope: vertical lines have an undefined slope, because the difference of any two $x$-values, the denominator of slope, is always 0 (and we can never divide by 0 !).



## Rate of Change

A rate of change compares a change in one quantity to a change in another quantity.

## Example:

The table to the right shows the distance a person walks for exercise.
Find the rate of change in distance with respect to time.
Rate of change $=\frac{\text { change in distance }}{\text { change in time }}=\frac{4.5-1.5}{90-30}=\frac{3}{60}=\frac{1}{20}=0.05 \mathrm{mi} / \mathrm{min}$

| Time <br> (minutes) | Distance <br> (miles) |
| :---: | :---: |
| 30 | 1.5 |
| 60 | 3 |
| 90 | 4.5 |

The slope of a line is a rate of change, as it measure the rate of change of the ordinates ( $y$ values) to the abscissas ( $x$-coordinates). We can graph real-world data to compare rates of change.

Example: Analyzing real-world data
A community theatre performed a play each Saturday evening for 10 consecutive weeks. The graph shows the attendance for the performances in weeks $1,4,6$, and 10. Describe the rates of change in
 attendance with respect to time.

| Weeks 1-4: <br> $m=\frac{232-124}{4-1}=\frac{108}{3}=36$ <br> people/week | Weeks 4-6: <br> $m=\frac{204-232}{6-4}=\frac{-28}{2}=-14$ <br> people/week | Weeks 6-10: <br> $m=\frac{72-204}{10-6}=\frac{-132}{4}=-14=33$ <br> people/week |
| :--- | :--- | :--- |

Interpretation: During the first four weeks, we see an increase in attendance of 36 people per week. During weeks 4-6 we see a decrease in attendance of 14 people per week. During the final weeks we see an even greater decrease in attendance.

## Example: Analyzing a graph.

The graph to the right shows a student commuting from home to school both by walking and taking the bus. Use the graph to describe the student's commute in words.

During the first part of the student's commute, we see an increase in distance, but it is not very steep in relation to time. The student must have been walking. Then the rate of change stops - there is no


Time (minutes) increase in distance as time increases. The student is not moving, so perhaps is waiting for the bus. Then we see a sharp increase in distance in relation to time, so the student is probably riding the bus at that point.

