Name: $\qquad$ Block: $\qquad$ Date: $\qquad$

## Direct Variation

When two things are related in such a way that the ratio of their values always remains the same, the two variables are said to be in direct variation.

| Direct Variation | Two variables show direct variation when they can be modeled by the <br> equation $y=a x$ |
| :--- | :--- |
| Constant of Variation $a \neq 0$. | The constant " $a$ " in $y=a x$ is called the constant of variation. |

Direct variation is sometimes written as $y=k x$, where $k=$ constant of variation.

## Example of Direct Variation:

Suppose an object on Earth weighs 150 pounds. On Neptune, the same object would weigh 165 pounds. We determine the weight on Neptune as compared to that on Earth by multiplying the object's weight on Earth by 1.1.
The weights on Earth and Neptune vary directly, and the constant of variation is 1.1.
So: $y=1.1 x$, where $x$ represents the weight on Earth, and $y$ represents the weight on Neptune.
Graphing: We graph direct variation following what we know about graphing slope-intercept form equations, $y=m x+b$. In direct variation equations, $m=a$, the constant of variation, and $b=0$.


## Key Concepts of Direct Variation

- $y=a x$, where $a=$ constant of variation
- $a$ is the slope of the graph of the line $(a=m)$
- The line always goes through the origin (since the $y$-intercept is always 0 )


## Identifying Direct Variation Equations

If a linear equation can be written in the form $y=a x$, then we have direct variation.
Do the following equations show direct variation? If so, what is the constant of variation?
(To answer these questions, solve for $y$ !)

| a) $y=4 x+2$ | b) $2 x-3 y=0$ | c) $-x+y=1$ | d) $4 x-5 y=0$ |
| :--- | :--- | :--- | :--- |
| Already in function | Put in function form: <br> $2 x-3 y=0$ |  |  |
| form; |  |  |  |
| Does not follow $y=a x$ <br> form, so, no, NOT $a$ <br> direct variation. | $-3 y=-2 x$ |  |  |

## Finding the Constant of Variation and Direct Variation Equation

Given the graph of a direct variation at the right, find the constant of variation and write the equation.

Since we know the model of a direct variation equation is $y=a x$, and we know a point on the graph ( $-1,2$ ), we can substitute the point coordinates
 for $y$ and $x$ :
$y=a x$
$2=a(-1)$
$a=-2$ (why?)
Another way is to find the slope, which is the constant of variation, using $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$; the second point would be the origin, $(0,0)$.
$m=a=\frac{2-0}{-1-0}=\frac{2}{-1}=-2$
Therefore, the direct variation equation is $y=-2 x$.
Since we always have a point through the origin with direct variations, we can find the constant of variation by finding a point and dividing $y$ by $x: a=\frac{y}{x}$.

## Word Problem 1:

The number s of tablespoons of sea salt needed in a saltwater fish tank varies directly with the number w of gallons in the tank. A pet shop owner recommends adding 100 tablespoons of sea salt to a 20 gallon tank.

- Write a direct variation equation that relates $w$ and $s$.
- How many tablespoons of salt should be added to a 30 gallon fish tank?


## Word Problem 2.

The table shows the cost $C$ of downloading s songs at an internet music site.
a) Explain why $C$ varies directly with $s$.
b) Write a direct variation that relates $s$ and $C$.

| Number <br> of songs, $s$ | Cost, $C$ <br> (dollars) |
| :---: | :---: |
| 3 | 2.97 |
| 5 | 4.95 |
| 7 | 6.93 |

