Factoring Quadratic Trinomials Notes

There are several ways we can factor a polynomial of the form $ax^2 + bx + c$, $a \neq 0$.

Method 1: Reverse FOIL.

Mentally work backwards from what we know about FOIL. This works best for the simple case, when $a = 1$. It is a lot harder when $a \neq 1$.

- List the factors for c.
- If the sign of c is positive, the signs of the binomial are the same. If c is positive, then the signs are positive; if c is negative, the signs are negative. We want the factors that ADD up to b.
- If the sign of c is negative, one of the binomials will have a positive sign, and the other will be negative. We want the factors that SUBTRACT to make b (we are still adding, but since they have opposite signs, we will essentially be subtracting; it is important to consider the sign when we add these numbers of different signs).

Example 1: Factor $x^2 + 5x + 6$

Step 1: List the factors of 6:

Step 2: The value of c, 6, is positive. Which factors of 6 add up to 5?

Step 3: The signs of the factors will be positive because b is positive.

Factored version: $(x + 3)(x + 2)$

Step 4: CHECK YOUR WORK. Multiply your answer and check it is what we started with.

Example 2: Factor $x^2 - 5x - 6$

Step 1: List the factors of 6:

Step 2: The value of c, -6, is negative. Which factors of 6 when subtracted give 5? Which factor should be negative and which should be positive?

Step 3: The signs of the factors will be positive because b is positive.

Factored version: $(x - 6)(x + 1)$

Step 4: CHECK YOUR WORK. Multiply your answer and check it is what we started with.

You try:

a) Factor $x^2 + 8x + 12$ b) Factor $x^2 - 10x + 10$ c) Factor $x^2 - 2x - 8$

Method 2: Box Method

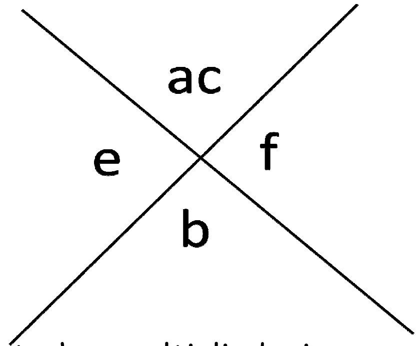
This method works well for any value of a.

- Draw a 2x2 box. Put ax^2 in the upper left box and c in the lower right box.
- Multiply a*c. List the factors. Based on the signs of the trinomial, we can determine whether we want factors that add up or subtract to c (see method 1).
- Place the factors as coefficients to the x-variable in the remaining boxes.
- Extract common factors vertically and horizontally. These are your binomials!

Example: $2x^2 - 5x - 3$

You try: $2x^2 - 7x + 12$

Method 3: Diamond Method



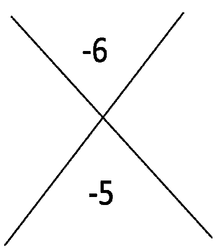
This method works for any value of a.

Steps:

- In the top, put the product of a and c.
- In the bottom, put the value of b.
- The left and the right locations are the numbers that when multiplied, give us ac, and when added, give us b, that is ef = ac and e + f = b.
- Put a fraction bar over the left and right values, and put a on top. Reduce.
- The top part of the fraction bar is the x-coefficient of the binomial factor, and the bottom part is the constant part.

Example: $2x^2 - 5x - 3$

1)



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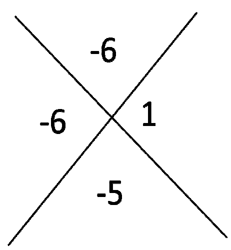
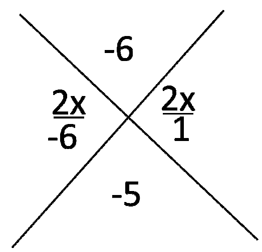


3)



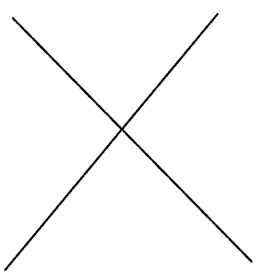
4)

$\frac{2x}{-6} = \frac{x}{-3} \longrightarrow x - 3$

$\frac{2x}{1} \longrightarrow 2x + 1$

Factored version: (x - 3)(2x + 1)…CHECK BY MULTIPLYING!

You try: $6x^2 - x - 2$



Method 4: Slide and Divide

This method works well for any value of a.

- "Slide" the leading coefficient,a, to the end, and multiply it by c. Pull out common factors, if any.
- Now we have the "simple" case, when a = 1. Factor using method 1.
- "Put back" the number you slid by dividing the number in each binomial by a.
- Simplify the fractions. If there is a denominator left in one of the binomials, make it the coefficient of the x-term for that binomial.

Example: Factor $3x^2 + x - 10$

Step 1: Slide a (3) to the end, multiplying by c (-10): $x^2 + x - 30$

Step 2: Factor using method 1: $(x + 6)(x - 5)$

Step 3: Divide numbers by a: $(x + 6/3)(x - 5/3)$

Step 4: Simplify fractions: $(x + 2)(x - 5/3)$

Step 5: Hey, there's a fraction left! Move the denominator in front of the coefficient:

$(x + 2)(3x - 5)$

Step 6: CHECK YOUR WORK. Multiply your resulting factors to check it is correct.

You try: Factor $2x^2 - 7x + 5$

Practice

Factor the following trinomials. You may use any method you wish, but try a few of them to help you find your favorite!

1) $x^2 + 6x + 5$

2) $x^2 - 4x - 12$

3) $x^2 - x - 12$

4) $p^2 + 9p + 14$

5) $2w^2 + 7w + 3$

6) $x^2 + 2x - 24$

7) $4x^2 - 4$

8) $5a^2 - 8a - 4$

9) $3n^2 + 13n + 4$

10) $-x^2 - 4x + 5$